BRIDGING EXEMPTION TEST
SEMESTER I, SESSION 2020/2021

COURSE	:	MATHEMATICS
PROGRAMME	:	FACULTY FOUNDATION UTM (BRIDGING)
DURATION	:	2 HOURS
DATE	:	OCTOBER 2020

INSTRUCTIONS TO CANDIDATE:

1. Answer all the questions.
2. All answers must be written in the answer booklet provided. Use a new page for each question.
3. The full marks for each question or section are shown in the bracket at the end of the question.
4. All steps must be shown clearly.
5. Only non-programmable and non-graphing scientific calculators can be used.
6. Answers may be given in the form of π, e, surd, fractions, or up to four significant figures, where appropriate, unless stated otherwise in the question.
7. You are not permitted to take the exam paper and the answer booklet(s) out of the exam hall.

QUESTION 1 (10 marks)

a) Given the polynomial, $P(x) = 6x^3 + 5x^2 - 7$ and the divisor, $D(x) = 3x^2 - 2x - 1$.

i) By using long division, find the Quotient and the Remainder and write the solution in the form of

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}.$$ (4 marks)

ii) Factorize $D(x)$ completely. (1 mark)

b) Without using calculator, solve the following simultaneous equation.

$$\begin{aligned} x + y &= 0 \\ 2x + 3y + 3z &= 1 \\ -x + y + z &= 1 \end{aligned}$$

(5 marks)

QUESTION 2 (10 marks)

Solve the following equations.

a) $2^x + 4^x = 2$. (5 marks)

b) $2\log_x 3 = 1 + \log_3 x$. (5 marks)

QUESTION 3 (10 marks)

a) Given $z = 4 - 3i$, find $\bar{z} + z(2i^{28} + i^{11})$ in the form $a + bi$. (3 marks)

b) Given $z = 3 - 5i$, find $\dfrac{z + 2i}{\bar{z}}$ in the form $a + bi$. (3 marks)

c) Given $z - 5 = i(10 + 4\bar{z})$. Find the value of z in the standard form where a and b are real numbers.

(4 marks)

QUESTION 4 (10 marks)

Given matrix $A = \begin{pmatrix} 2 & q & 3 \\ 5 & -1 & 7 \\ r-3 & r+s & 1 \end{pmatrix}$.

a) If A is a symmetric matrix, find the values of q, r and s. (4 marks)

b) i) If $q = 1$, $r = 2$ and $s = -1$, show that A^{-1} is exist.
(2 marks)

ii) Find A^{-1} using adjoint method.
(4 marks)

QUESTION 5 (10 marks)

a) Two functions are defined as $f(x) = 2 - x^2$ and $g(x) = \sqrt{2x-4}$. Find

i) $(f \circ g)(x)$. (2 marks)

ii) Determine the domain for $g(x)$. (1 mark)

iii) $g^{-1}(x)$. (2 marks)

b) Find the centre and vertices for the equation

$$\frac{(x+2)^2}{25} + \frac{(y-4)^2}{4} = 1.$$

Hence, sketch the graph of the equation and identify the shape.
(5 marks)

QUESTION 6 (10 marks)

a) Given four points

$$A(4,3,-3),\ B(1,5,-2),\ C(0,2,3) \text{ and } D(4,3,3).$$

i) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$. (4 marks)

ii) Find the parametric equation of a line that passes through the points A and C.

(3 marks)

b) Given two vectors **u** = **i** + 3**j** and **v** = 2**i** + 2**j** + 5**k**. Find the area of a parallelogram bounded by the vectors **u** and **v**. (3 marks)

QUESTION 7 (20 marks)

a) Use the definition of derivative to find $f'(x)$ for the function $f(x) = 2 - 3x + 2x^2$.

(5 marks)

b) Find $\dfrac{dy}{dx}$ for $y \ln x^4 + \cos(2y) = x^3 y^2$ by using implicit differentiation.

(5 marks)

c) If $x = 2\cos t - \cos 2t$ and $y = 2\sin t - \sin 2t$, find $\dfrac{dy}{dx}$. Hence, evaluate $\dfrac{dy}{dx}$ when $t = \dfrac{\pi}{2}$.

(5 marks)

d) Given $y = e^{2x}\cos x$. Show that $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 0$. (5 marks)

QUESTION 8 (20 marks)

a) Use integration by part to evaluate

$$\int 4x\cos(2-3x)\,dx.$$

(5 marks)

b) By using appropriate substitution, evaluate

$$\int_0^1 \frac{x}{(x+1)^3}\, dx.$$

(5 marks)

c) Given that $y = \frac{x^2+1}{x+4}$ and $\frac{dy}{dx} = f(x)$. Find

$$\int_0^1 [x + 2f(x)]dx$$

(5 marks)

d) By using partial fraction, evaluate the integral

$$\int_2^3 \frac{6-3x}{x^2+x-2}dx.$$

(5 marks)

List of Mathematical Formulae

Trigonometry

- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \operatorname{cosec}^2 x$
- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $\quad = 2\cos^2 x - 1$
  $\quad = 1 - 2\sin^2 x$
- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
- $2\sin x \cos y = \sin(x + y) + \sin(x - y)$
- $2\sin x \sin y = -\cos(x + y) + \cos(x - y)$
- $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

Conic Sections

- Circle with centre (h,k) and radius r:
  $$(x-h)^2 + (y-k)^2 = r^2.$$
- Parabola with vertex (h,k), focus $(h+p,k)$ and directrix $x = h - p$:
  $$(y-k)^2 = 4p(x-h).$$
- Ellipse with centre (h,k) and foci $(h \pm c,k)$ where $c^2 = a^2 - b^2$:
  $$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vectors

- Dot (scalar) product.
  $$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta,$$
  where
  $\mathbf{u} \cdot \mathbf{v} = (x_1\,\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}) \cdot (x_1\,\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k})$

- Cross product.

  If $a = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $b = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$, then $a \times b = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

  If $\vec{OA} = a$ and $\vec{OB} = b$, then

  i) The area of the parallelogram OACB is $\left|\vec{OA} \times \vec{OB}\right| = |a \times b|$

  ii) The area of the triangle OAB is $\dfrac{1}{2}\left|\vec{OA} \times \vec{OB}\right| = \dfrac{1}{2}|a \times b|$

Logarithm

- $a^x = e^{x \ln a}$
- $\log_a x = \dfrac{\log_b x}{\log_b a}$

Differentiations	Integrations
$\dfrac{d}{dx}[k] = 0, k$ constant	$\int k\ dx = kx + C,\ k\ constant$
$\dfrac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C, n \neq -1$
$\dfrac{d}{dx}[e^x] = e^x$	$\int e^x dx = e^x + C$
$\dfrac{d}{dx}[\ln x] = \dfrac{1}{x}$	$\int \dfrac{dx}{x} = \ln\lvert x\rvert + C$
$\dfrac{d}{dx}[\cos x] = -\sin x$	$\int \sin x dx = -\cos x + C$
$\dfrac{d}{dx}[\sin x] = \cos x$	$\int \cos x dx = \sin x + C$
$\dfrac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\dfrac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x dx = -\cot x + C$
$\dfrac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\dfrac{d}{dx}[\operatorname{cosec} x] = \operatorname{cosec}\ x\ \cot x$	$\int \operatorname{cosec}\ x \cot x dx = -\operatorname{cosec} x + C$

Parametric Differentiations

- $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$
- $\dfrac{d^2 y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \times \dfrac{dt}{dx}$

Integration by parts

- $\int u\ dv = uv - \int v\ du$