QUESTION 1 (10 MARKS)

a) Given the polynomial $P(x) = 2x^3 + x^2 - 4x + 7$ and the divisor, $D(x) = x^2 + 2x - 8$.

i) Use long division to find the Quotient and the Remainder. Write the solution in the form of

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}.$$

(4 marks)

ii) Hence, factorize $D(x)$ completely.

(1 mark)

b) Solve the following system of equations without using calculator.

$$\begin{aligned} 2x + y + z &= 4 \\ -\frac{3}{2}x - z &= -5 \\ x - 2y + 3z &= 7. \end{aligned}$$

(5 marks)

QUESTION 2 (10 MARKS)

Solve the following equations.

a) $4^{2x+1} - 4^{x+2} + 2 = 0$.

(5 marks)

b) $\log_3 3x = \log_9 (3x+2)$.

(5 marks)

QUESTION 3 (10 MARKS)

a) Given $z = 1 + 3i$ and $\overline{z}$ is the conjugate of z, express $z - 2\overline{z}\left(\overline{z} - i^{21}\right)$ in the form $a + bi$.

(3 marks)

b) Given $\overline{z} = -2+3i$, express $\dfrac{\overline{z}^2}{z-\overline{z}}$ in the form $a+bi$.

(3 marks)

c) The complex numbers of w and z are given by $w = 3-4i$ and $z = \dfrac{2w+3i}{iw+3}$. Express z in the form $a+bi$.

(4 marks)

QUESTION 4 (10 MARKS)

a) Given the matrix $A = \begin{pmatrix} 1 & 2 & -4 \\ 4 & 1 & 3 \\ 3 & -1 & 2 \end{pmatrix}$.

i) Find M_{31} and C_{32}.

(3 marks)

ii) Hence, find the determinant of matrix A.

(2 marks)

b) Given that $B = \begin{pmatrix} 1 & -7 & 4 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \end{pmatrix}$. Find B^{-1} using adjoint method.

(5 marks)

QUESTION 5 (10 marks)

a) Given $f(x) = x^2 + 3$ and $g(x) = x + 2$.

i) State the domain of $\dfrac{f(x)}{g(x)}$,

(2 marks)

ii) Find $f \circ g^{-1}(x)$.

(3 marks)

b) Write the equation of the circle $x^2 + y^2 - 14x + 6y + 49 = 0$ in the standard form $(x-h)^2 + (y-k)^2 = r^2$. Hence, sketch the circle with the correct center and radius.

(5 marks)

QUESTION 6 (10 marks)

a) Given three vectors, $\mathbf{a} = \langle 2, -1, 4 \rangle, \mathbf{b} = \langle 1, 2, -3 \rangle$ and $\mathbf{c} = \langle 5, -1, 1 \rangle$

i) Determine $\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})$.

(4 marks)

ii) Show the two vectors, $\mathbf{b}$ and $\mathbf{c}$ are perpendicular to each other.

(2 marks)

b) Find the equation of the line that passes through the points $A(6, -4, 7)$ and $B(1, 2, 2)$. Express your answer in Cartesian form.

(4 marks)

QUESTION 7 (20 marks)

a) Find $f'(x)$ for the function of $f(x) = x^2 - 5x$ using definition of derivative.

(5 marks)

b) Using implicit differentiation, find $\dfrac{dy}{dx}$ for the equation $x^2 y = \ln(x+y) + e^y$.

(6 marks)

c) Given $y = 3e^{x^3}$, show that $\dfrac{d^2y}{dx^2} - 3x^2 \dfrac{dy}{dx} - 6xy = 0$.

(4 marks)

d) Given the parametric equations:

$$x = t^2 + 3 \qquad \text{and} \qquad y = \frac{t^2+1}{t^2-1}.$$

Find $\frac{dy}{dx}$. Hence, evaluate $\frac{dy}{dx}$ when $t = 3$.

(5 marks)

QUESTION 8 (20 MARKS)

a) Use integration by parts to evaluate

$$\int_3^5 2x\,e^x\,dx$$

(5 marks)

b) By using appropriate substitution, evaluate

$$\int \frac{3x+5}{(x+2)^2}\,dx.$$

(5 marks)

c) Evaluate $\int 5x^3 \cos 3x\,dx$ using tabular method.

(5 marks)

d) By using partial fractions, show that $\int \frac{3x+1}{(x+2)(2x-1)}\,dx = \ln(x+2)\sqrt{2x-1} + C$.

(5 marks)

List of Mathematical Formulae

Trigonometry

- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \operatorname{cosec}^2 x$
- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 2\cos^2 x - 1$
  $= 1 - 2\sin^2 x$
- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
- $2\sin x \cos y = \sin(x + y) + \sin(x - y)$
- $2\sin x \sin y = -\cos(x + y) + \cos(x - y)$
- $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

Conic Sections

- Circle with centre (h,k) and radius r:
  $$(x-h)^2 + (y-k)^2 = r^2.$$
- Parabola with vertex (h,k), focus $(h+p,k)$ and directrix $x = h - p$:
  $$(y-k)^2 = 4p(x-h).$$
- Ellipse with centre (h,k) and foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$:
  $$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vectors

- Dot (scalar) product.

  $\mathbf{u} \,.\, \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta,$ where $\mathbf{u} \,.\, \mathbf{v} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$.

- Cross product.

  If $a = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $b = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$, then $a \times b = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

If $\vec{OA} = a$ and $\vec{OB} = b$, then

i) The area of the parallelogram OACB is $\left|\vec{OA} \times \vec{OB}\right| = |a \times b|$

ii) The area of the triangle OAB is $\frac{1}{2}\left|\vec{OA} \times \vec{OB}\right| = \frac{1}{2}|a \times b|$

Logarithm

- $a^x = e^{x \ln a}$
- $\log_a x = \dfrac{\log_b x}{\log_b a}$

Differentiations	Integrations
$\frac{d}{dx}[k] = 0, k$ constant	$\int k\ dx = kx + C,\ k\ constant$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}[\ln x] = \frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x dx = -\cot x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}[\operatorname{cosec} x] = \operatorname{cosec}\ x \cot x$	$\int \operatorname{cosec}\ x \cot x dx = -\operatorname{cosec} x + C$

Parametric Differentiations

- $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$
- $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \times \dfrac{dt}{dx}$

Integration by parts

- $\int u\ dv = uv - \int v\ du$